DIGITAL IMAGE PROCESSING AND ENHANCEMENT
Lecture 16 — Session 1, 2009
Imaging Tomography

DIP&E-16.1 Imaging Tomography

In the previous lecture we looked at aperture synthesis radiastonomy, which is a form of tomography.

Most of you will have heard of imaging tomography.

- *The word CAT scan, stands for "Computed Aided Tomography".*

In fact, many people may have already had either a CAT scan or an MRI scan.

- *MRI stands for "Magnetic Resonance Imaging", which is more accurately called Nuclear Magnetic Resonance Imaging.*

"Nuclear" was dropped because it made people think of is nuclear radiation.

In fact, MRI involves only a very strong magnetic field and microwave radiation.

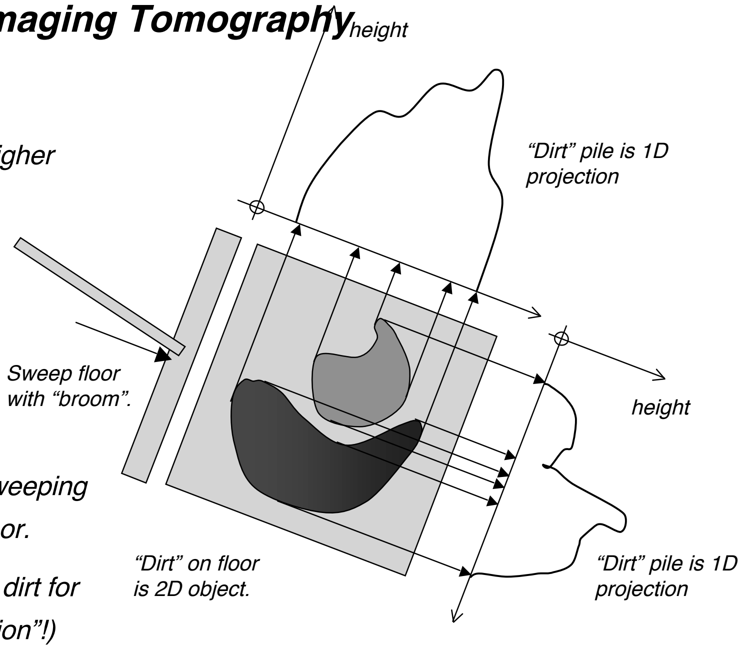
To begin with, we will look at a very simple-minded analogy — dirt on a floor!

DIP&E-16.2 Imaging Tomography

Tomography is the reconstruction of a higher dimensional object from its multiple projections on a lower dimension.

For example:

- *Take multiple projections by sweeping broom across floor.*
- *(But leave some dirt for the next "projection"!)*

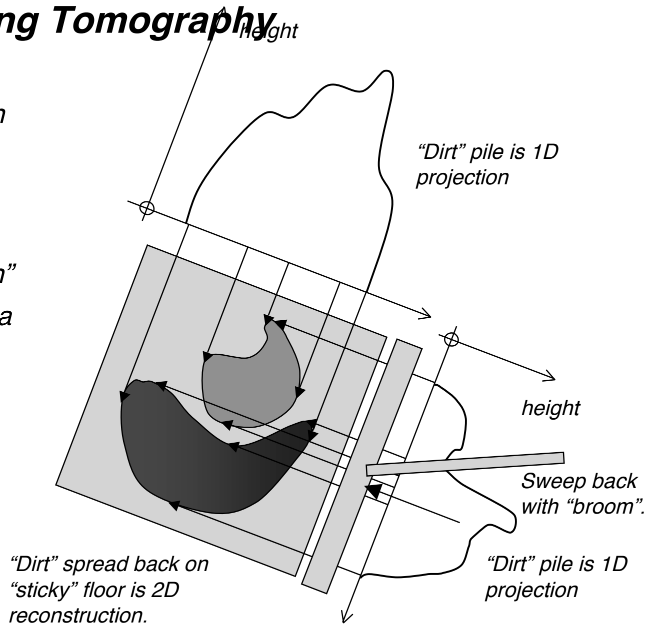


DIP&E-16.3 Imaging Tomography

Tomographic reconstruction is the opposite operation:

For example:

- *Sweep each "projection" pile of dirt back across a "sticky" floor.*
- *It will not be perfect, due to some dirt adhering in places where there should be none.*



DIP&E-16.4 Imaging Tomography

Central slice theorem relates projections to central slices of Fourier domain.

- *Along the horizontal axis, a projection of a 2D object is:*

$$f_p(y) = \int f(x,y)dx$$

and, its 1D Fourier transform is:

$$F_p(v) = \int f_p(y)\exp(-j2\pi vy)dy$$
$$= \iint f(x,y)\exp(-j2\pi vy)dxdy,$$

where $j = \sqrt{-1}$.

But the 2D Fourier transform of the object is:

$$F(u,v) = \iint f(x,y)\exp[-j2\pi(ux+vy)]dxdy$$

which is the same as F_p *for u = 0 — i.e., a "central slice" of the 2D F.T.*

DIP&E-16.5 Imaging Tomography

The central slice for u = 0 is easily generalised to any central slice in the Fourier domain, by rotation of axes:

- *Rotating the object plane through an angle* θ *to new axes x',y', gives a projection along the new horizontal axis:*

$$f_p(y') = \int f(x',y')dx',$$

and

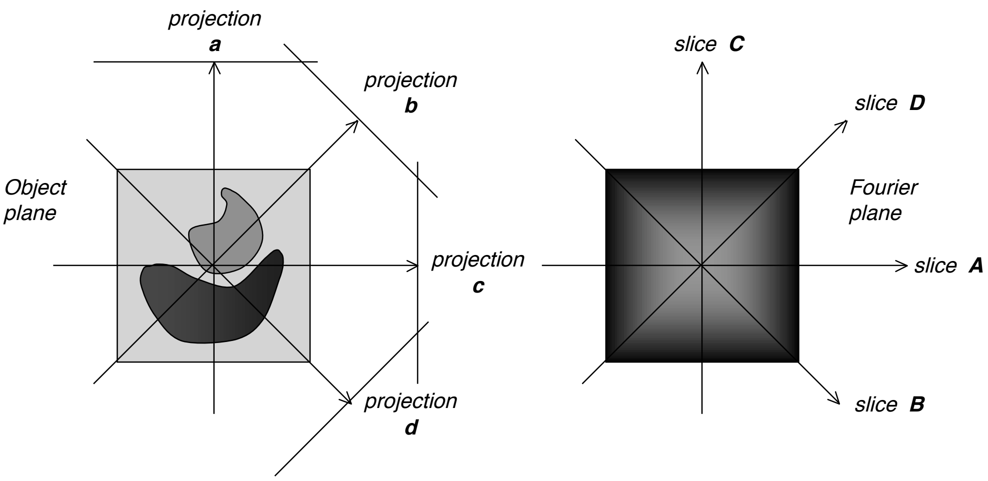
$$F_p(v') = \int f_p(y')\exp(-j2\pi v'y')dy'$$
$$= \iint f(x',y')\exp(-j2\pi v'y')dx'dy'.$$

But rotation through θ *in the object domain is equivalent to rotation through* θ *in the Fourier domain,*

- *thus* $F_p(v')$ *is a "central slice" through the 2D Fourier transform for u' = 0,*
  - *i.e., at an angle* $-\theta$.

DIP&E-16.6 Imaging Tomography

How is the central slice theorem useful?



DIP&E-16.7 Imaging Tomography

Using the central slice theorem, take 1D Fourier transform of each projection,

- *Place result in 2D Fourier plane along corresponding slice,*
- *Gradually build up filled 2D Fourier plane.*
- *Inverse 2D Fourier transform yields reconstructed 2D object.*

By analogy, the same increase in dimensions, and technique applies for 3D reconstruction from 2D projections.

- *Thus, X-ray CAT scan (computer aided tomography) yields 3D view of human body through multiple 2D X-ray projections (shadowgraphs — like usual X-ray picture).*

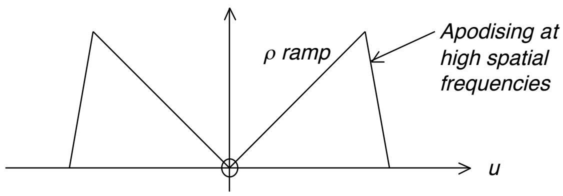
DIP&E-16.8 Imaging Tomography

Alternatively, we can apply "back projection" reconstruction.

- *Back projection is the equivalent of the sweep-broom across the sticky floor reconstruction, described in the "dirty floor" analogy.*

Problem — to overcome the fill-in effect of back-projecting across the full image.

- *Solution: pre-filter the projections to ramp up high spatial frequencies.*
- *Result: is known as the Radon transform.*



DIP&E-16.9 Imaging Tomography

Applications:

- *So called "CAT" scanners (Computer Aided Tomography)*
  - *This name has become associated with X-ray tomography*
- *MRI (Magnetic Resonance Imaging) devices*
  - *Should be called Nuclear Magnetic Resonance Imaging, but "nuclear" considered a dangerous word!*
  - *Actually, nothing to do with radioactive nuclei*
- *PET (Positron Emission Tomography)*
  - *Does use radio-isotopes; detect emissions outside the body*

DIP&E-16.10 Imaging Tomography

CAT Scanners (X-ray tomography):

- *X-ray “shadow-graphs” (the usual projection X-ray “picture”) taken from many different angles, as patient rotated;*
- *These are combined,*
  - *either by back-projection,*
  - *or through the Fourier domain (Central Slice theorem).*
- *Oldest medical tomographic method (has been around since 1960s)*
  - *Resolution not the greatest*
  - *Possible health danger due to large X-ray dose*

DIP&E-16.11 Imaging Tomography

MRI Scanners (nuclear magnetic resonance imaging tomography):

- *Depends on spin of hydrogen nuclei*
  - *In a magnetic field, these tend to line up with the field*
  - *Precess at a certain frequency in presence of microwave energy*
  - *If field has gradient, then precession frequency varies across gradient*
  - *Can use this to detect density of hydrogen nuclei with position*
- *Newer medical tomographic method, since the late 1970s*
  - *Resolution much better than CAT scanners (though different tissues)*
  - *No health danger of X-ray sort, but needs very high magnetic fields*

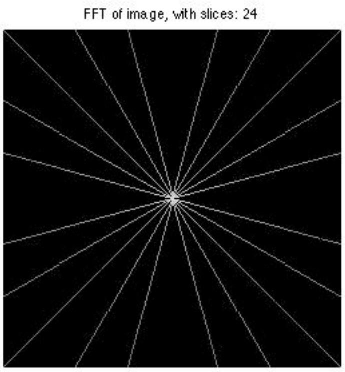
DIP&E-16.12 Imaging Tomography

PET Scanners (positron emission tomography):

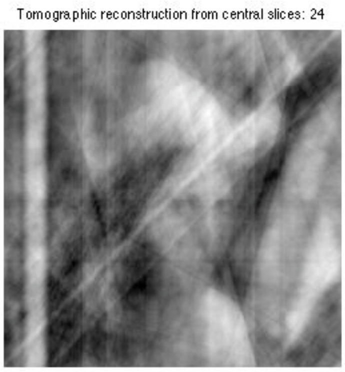
- *Patient given sugar in which normal carbon atoms replaced with a radio-isotope of carbon*
  - *Assumes greatest uptake of sugar in cells working "hardest"*
  - *External detection of radio-active emission*
  - *Form tomographic picture of hard-working cells*
- *Special medical or scientific tomographic method, since the 1980s*
  - *Used, e.g., to map areas of the brain under different stimulation*
  - *Possible health danger due to radio-active isotopes*

DIP&E-16.13 Imaging Tomography

Example of central slice theorem applied to Lena in MATLAB:



FFT of Lena, central-slice masked



IFFT, tomographic reconstruction